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APICAL VORTICES AND TRUMPET-SHAPED VORTEX SHEETS

By Maurice Roy

Translation

"Sur la théorie de l'aile en delta - Tourbillons d'apex et nappes en cornet." La Recherche Aéro., No. 56, Feb. 1957, pp. 3-12.

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ABSTRACT

The general problem of flow phenomena on thin delta wings is discussed, with the purpose of establishing the experimental background for development of a suitable theory. Some general equations for the trumpet-shaped vortex sheet are developed, but no solutions are attempted. A simplified approach, representing the trumpet-shaped sheet with a continuous distribution of vorticity and with sources and sinks in the flow is suggested. Attempts at calculation of the flow using the simplified approach are currently in progress.

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APICAL VORTICES AND TRUMPET-SHAPED VORTEX SHEETS[1]

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INTRODUCTION

For some time past numerous papers have appeared with the aim of establishing, for the delta wing and wing with large sweepback, a simplified theory that would be in harmony with the increasingly known features of the flow of the air around wings of this kind.

It is essentially with the general outline, or scheme of this flow, with a view toward employing it as a basis for a theory, that I shall here concern myself.

Since the observation of phenomena must necessarily precede the elaboration of their theory, I have for the past six years at least given great importance to investigations on the visualizations of flows. By this method and thanks especially to efforts and the ability of Werle, who among others has applied the technique of milk filaments, white or colored, the hydrodynamic tunnel built at Chatillon according to my conception has been intensively utilized and has furnished valuable cross-checking of other tests conducted in the wind tunnel and accompanied by visualization through smokes, liquid coatings, and wool filaments.

In the hydrodynamic tunnel in particular it has been possible to enter more deeply into details that otherwise escape observation, and Werle has already presented twice in <u>La Recherche Aéronautique</u> results of various studies of this kind (ref. 1).

I underline the fact here that it is by no means unknown that in these studies there is considerable divergence between the Reynolds numbers realized in these wind tunnel tests and those relative to a real airplane wing, even if the latter flies only at velocities where the compressibility of the air is legitimately neglected. But, the necessary changes having been made, these tests furnish qualitative information on these phenomena which, with some humor, might be called as "illuminating" as the intensely bright colored filaments, in the tests under consideration.

[1]"Sur la théorie de l'aile en delta - Tourbillons d'apex et nappes en cornet." La Recherche Aéro., No. 56, Feb. 1957, pp. 3-12.

SCHEME OF THE CONTINUOUS POTENTIAL

The delta wing here considered consists of an angular plane infinite sector, of vertex angle $\pi - 2\varphi$, φ being the sweepback angle.

With respect to the x,y,z-axes attached to the wing (fig. 1), the x-axis being the longitudinal axis of the wing oriented downstream, the flow is assumed steady and of velocity at infinity V_0 under angle of attack α and without side slip. The fluid is assumed incompressible and perfect.

The components u,v,w of the relative motion are put in the nondimensional form

$$\left.\begin{aligned} u/V_0 \cos\alpha &= 1 + \tilde{\omega} \\ v/V_0 \cos\alpha &= \chi \\ w/V_0 \cos\alpha &= \tau \end{aligned}\right\} \qquad (1)$$

We assume that the flow is conical with respect to the apex ($\tilde{\omega}$, χ, and τ functions only of y/x and z/x) and that in the transverse plane with complex variable

$$\xi \equiv \eta + i\zeta$$

$$\eta = \frac{y}{x \cot\varphi}$$

$$\zeta = \frac{z}{x \cot\varphi}$$

the flux is solenoidal, assumptions that rigorously are not compatible, as will be seen further on. The reduced velocity $(1 + \tilde{\omega}, \chi, \tau)$ can then be defined from a complex potential $f(\xi)$ such that

$$\left.\begin{aligned} \chi - i\tau &\equiv df/d\xi \\ \tilde{\omega} &= \cot\varphi \cdot \mathcal{R}(f - \xi df/d\xi) \end{aligned}\right\} \qquad (2)$$

The scheme of a continuous potential implies, in the transverse plane, a flow with passage around the extremities $\eta = \pm 1$ of the rectilinear cut (-1, +1) of the η-axis that represents the cross section of the wing whose span, at the height x, is $2b \equiv 2x \cot \varphi$.

In the cut half-plane $\eta > 0$, where $\xi \equiv re^{i\theta}$, θ varying from $-\pi/2$ to $\pi/2$, the potential in question $f(\xi)$ and the reduced velocity are given by

$$f(\xi) = - i \sqrt{\xi^2 - 1} \tan \alpha$$

$$\chi - i\tau = - i(\xi/\sqrt{\xi^2 - 1}) \tan \alpha$$

$$\tilde{\omega} = \mathcal{R}(i/\sqrt{\xi^2 - 1}) \tan \alpha \cot \varphi$$

The lower and upper surfaces of the wing, indicated by e and r, are respectively defined by $0 \leq \eta_{ei} \leq 1$, $\zeta_{ei} \equiv 0\pm$, and the reduced velocities there take the values

$$\tilde{\omega}_{ei} = j_{ei} \frac{\tan \alpha \cot \varphi}{\sqrt{1 - \eta^2}}$$

$$\chi_{ei} = - j_{ei} \frac{\eta \tan \alpha}{\sqrt{1 - \eta^2}}$$

$$\tau_{ei} = 0$$

with

$$j_{ei} \equiv \pm 1$$

From this it follows that on the plan form of the wing the tangent to the wall streamline of the upper or lower surface is inclined to the x-axis by an angle β (see fig. 2), so that

$$\tan \beta_{ei} = \frac{\chi_{ei}}{1 + \tilde{\omega}_{ei}} = \frac{- \eta}{\cot \varphi + j_{ei} \sqrt{1 - \eta^2} \cot \alpha}$$

On the upper surface the wall streamlines, or the limit lines of the flow, start out orthogonal to the leading edge and flow toward infinity downstream, becoming asymptotically parallel to the x-axis. On the lower surface, however, all the limit lines start out from the apex O. Some strike the leading edge orthogonally, and others flow toward infinity downstream becoming asymptotically parallel to the x-axis. These two categories separate along a rectilinear border inclined to the x-axis by the angle β_l so that $\tan \beta_l = y/x = \eta \cot \varphi$; that is,

$$\tan \beta_l = \cot \varphi \sqrt{1 - \left(\frac{2 \tan \alpha}{\sin 2\varphi}\right)^2}$$

This line of division of the wall flux on the lower surface does not exist; therefore ($\tan \beta_l > 0$) unless α is sufficiently small so that $\tan \alpha$ remains less than $(\sin 2\varphi)/2$, a limit that attains its maximum for $\varphi = 45^{\circ}$, the intermediate case where the strong sweepbacks ($\varphi > 45^{\circ}$) and the small sweepbacks ($\varphi < 45^{\circ}$) meet. The large angles of attack are of course here out of the question because the theories of lift and pressure distribution over the wings envisage only sufficiently small angles of attack.

In any case, the rolling up of the wall lines of the lower surface at the leading edge corresponds to a flowing around the leading edge by the adjacent fluid orthogonally to the leading edge and with an infinite velocity; hence with an infinite negative pressure.

In this scheme the continuous potential therefore implies that around the leading edge and in a plane locally perpendicular to it the irrotational flow of the incompressible and perfect fluid is analogous to that of a plane stream flowing around the edge of a thin plane under nonzero angle of attack. Hence there are again obtained the singularities or 'physical aberrations' of the theory of the rectilinear wing profile and the necessity of a theoretical suction effect on the leading edge; whereas in reality a detachment or separation of the flow would take place along this leading edge.

After reattaching on the upper surface the flux that rolls up from the lower surface by flowing around orthogonally to the leading edge would embody a practically stationary eddy zone along this edge, a zone that would thus constitute a vortex roll along the leading edge on the upper surface. It is to such a roll that the British term 'bubble' would in this case appear to correspond.

Figure 3 briefly represents these characteristics of the flow envisaged.

APICAL VORTICES AND TRUMPET-SHAPED VORTEX SHEET

The preceding scheme of the continuous potential has been envisaged above because it is classical and in order to bring out the fundamental features, which are very simple. But it does not correspond to reality except perhaps for very small angles of attack, a case for which it may even be doubted whether it is of sufficient interest to consider. We shall therefore preferably seek a scheme of a greater validity.

For a plane thin delta wing with sufficiently sharp leading edges, the flow around the anterior part of the wing, the immediate neighborhood of the apex being however probably excepted, is very similar to the flow relative to the indefinitely extended delta wing.

At the O.N.E.R.A. the great number of wind-tunnel tests that have been conducted in 1950-51 on wings with strong sweepback and with plan form more or less similar to the delta have brought into evidence the formation of vortex zones developing above the wing and starting from the apex, zones rather clearly characterized as soon as the angle of attack reaches 6° to 9° approximately for backsweep angles above 45°.

In 1951-52, and as I have pointed out above, I made considerable effort to visualize by various means the flows in question, in air and in water. Apart from the particular phenomena that appear near the points or marginal extremities of these wings and in certain regions of the trailing edge, the system of vortices of the upper surface appeared, for several observers, to consist of two symmetrical vortices, issuing approximately from the apex and forming, above the upper surface, a V situated in a plane less inclined to that of the wing than the velocity at infinity and less open than the V formed by the rectilinear leading edges.

Remarking that these vortices can only be fed and strengthened progressively by the ambient flow, I derived from the entire series of tests the schematization of the flow in question by two trumpet-shaped sheets according to the scheme of figure 4.

Some recent publications (ref. 2) use schemes that appear to present a rather striking analogy with that of my 'trumpet vortex sheets.' I ought therefore to mention, as regards the question of priority, that it was in 1952 that I set forth this conception in the following terms (ref. 3): "The two principal apical vortices appear to me to arise, for each half wing, from the rolling up of a vortex sheet detaching from the upper surface of the wing into a 'trumpet,' approximately orthogonally to the surface and along an almost straight line starting from the apex, a line which is more or less near the geometric leading edge and constituting the trace on the upper surface of a vortical 'bulkhead' between the two flows of the upper and lower surfaces."

The motion of these flows was described as follows: The upper-surface flow passing above the apex of the 'raised arrow' which constitutes the wing studied, warps laterally over the back of the wing in passing along the streams, likewise deviated laterally by the sweepback, the flow of the lower surface causing these streams to roll up toward the upper surface in flowing around the leading edge.

The distance apart between the line of detachment (or separation) seen above, or the starting-out line of the trumpet sheet, and the geometric line constituted by the leading edge is a function of the radius of curvature of the profile at this leading edge. In particular, this distance decreases as the radius of curvature decreases (fig. 5), and it tends to zero when the wing profile thins down to a line, the leading edge becoming sharp and preferably tapered.

This conception has recently been evoked in several publications, notably in La Recherche Aéronautique by R. Legendre (ref. 4) and by H. Werle (ref. 5). An almost identical conception has been adopted by C. E. Brown and W. H. Michael in a very interesting paper (ref. 6), in referring to the work of R. Legendre, but without mentioning the origin of my scheme of a trumpet sheet.

In order to illustrate what was said above and as a simple example, figure 6 presents the visualization by milk filaments of the flow on the lower surface ($\alpha = 20^{\circ}$) and on the upper surface ($\alpha = 11^{\circ}$) of a plane, thin, delta wing ($\varphi = 60^{\circ}$) with sharp leading edge. The dividing of the wall lines on the lower surface and the rolling up of the trumpet sheet on the upper surface are particularly recognizable in these two pictures.

PSEUDO CROSSFLOW

Let us consider the tangent lines in a transverse plane $x = \text{constant}$ with the velocity components situated in this plane. They are usually treated as streamlines of a crossflow. In fact it is a question of a pseudo crossflow, in the sense that the flux is not solenoidal.

In fact, in an incompressible fluid, as is here envisaged, the divergence of the transverse velocity is not zero, since we have

$$\partial v/\partial y + \partial w/\partial z = - \partial u/\partial x$$

and $\partial u/\partial x$ cannot be identically zero in the assumption of conicity.

Let us verify this by computing $\partial u/\partial x$.

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We shall use the notations previously defined:

$$u/V_0 \cos \alpha \equiv 1 + \tilde{\omega}(\eta,\zeta)$$

$$v/V_0 \cos \alpha \equiv \chi(\eta,\tau)$$

$$w/V_0 \cos \alpha \equiv \tau(\eta,\zeta)$$

$$\eta \equiv \frac{y}{x \cot \varphi}$$

$$\zeta \equiv \frac{z}{x \cot \varphi}$$

We have, in the entire irrotational flux (i.e., outside the vortex sheets or concentrated vortices)

$$\frac{\partial u}{\partial x} = \frac{V_0 \cos \alpha \cot \varphi}{x} \left[\eta^2 \frac{\partial \chi}{\partial \eta} + \eta\zeta\left(\frac{\partial \chi}{\partial \zeta} + \frac{\partial \tau}{\partial \eta}\right) + \zeta^2 \frac{\partial \tau}{\partial \zeta}\right] \tag{3}$$

an expression in which it is seen that the second member cannot be generally and throughout nonzero, nor even necessarily negligible in the approximate theories.

This second term, compared with the divergence of the transverse velocity $\partial v/\partial y + \partial u/\partial z$ is of the order of $\cot^2\varphi$, so that it approaches more nearly zero as φ approaches $\pi/2$.

From this it may also be concluded that in an incompressible fluid the pseudo crossflow requires a distribution, in its plane, of sources and sinks. In a certain approximate way it can be imagined that sources and sinks are everywhere negligible and this is the way, in fact, that has up to now been followed by most authors.

For my part, and as I have expressed it in 1953 at Göttingen, at the annual meeting in this town of the Wissenschaftliche Gesellschaft für Luftfahrt, I consider on the contrary that a more acceptable scheme of a pseudo crossflow should involve some distribution, at least concentrated, of sources and sinks. In my communication of that time, which could not be published for incidental reasons, I had schematized this distribution by combining two sinks with the two underside vortices and by adding a compensating source situated in the plane of symmetry z,x of the flow.

It can be considered that sources (or sinks), which correspond to the fact that $\delta u/\delta x$ is not rigorously zero in the entire transverse plane, are throughout negligible, or that their over-all effect, even in the neighborhood of the wing, is equivalent to that of a certain distribution of sources (or sinks) concentrated at certain points or on certain lines. In any case, if one assumes the existence of a sheet, conical because of the conical affinity assumed for the entire flow, of free vortices, it will be shown below that the section of such a sheet in the transverse plane ζ should be represented by a line of vortex sinks (or sources).

Here we may remark only that the rolling up, so clearly revealed by experiment, of the lower-surface flow along the sharp leading edge which we assume, necessitates in the plane ξ of the pseudo crossflow the existence of a border line (L) passing at a distance from the leading edge and ending on the upper surface, or on the positive part of the ζ-axis, and a stagnation point A_e (fig. 7).

The area V comprised between the η-axis and the line (L), on the side $\zeta > 0$, is fed by the 'strait' bounded by the points A_1 and A_1' of the η-axis, and receives an effective amount that should then be absorbed by the sinks, distributed or concentrated, in the area V. In compensation of these sinks it is necessary to conceive in the plane ξ and outside the area V the existence of equivalent sources, distributed or concentrated. In fact, when one is concerned especially with evaluating with a suitable approximation the velocities and pressures on the wing and in its neighborhood it is not excluded in advance that the compensating sources in question are transferred to infinity, the regularity of the potential and of the reduced velocity then being given up.

Moreover, as will be seen, the trumpet sheet and the conical distribution of velocity that it implies necessarily associate surface sources or sinks with the surface vortices that constitute the sheet.

EQUATION OF THE TRUMPET VORTEX SHEET

In the plane ξ (fig. 8) and over the cross section of the sheet emanating from the leading edge to the right A_1 $(\eta = +1)$, we denote by s the curvilinear abscissa of the current point (M starting from A_1) and by $\vec{s}$ and $\vec{n}$ two axes oriented along the directed tangent and following the normal to the curvilinear section in question, the plane (s,n) being itself oriented in the same sense as the plane (η,ζ).

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We denote with indices e and i the two faces of the sheet that prolong the upper surface and lower surface of the wing and put at the current point M of the sheet:

$$\left.\begin{aligned} G(s) &= \Phi_e - \Phi_i \\ G'(s) &\equiv dG/ds = \nu_{es} - \nu_{si} \\ \delta(s) &= \nu_{n_e} - \nu_{n_i} \end{aligned}\right\} \quad (4)$$

V_s and V_n denoting the components along the s- and n-axes of the reduced velocity (χ,τ), itself derived from the velocity potential $\Phi(\eta,\zeta)$.

According to this definition, for a positive element ds of the section of the sheet, $(-G'\ ds)$ represents the direct circulation about the element ds, and $(\delta\ ds)$ represents the volume of fluid emitted algebraically by the element ds of a conical segment of the sheet whose height along the x-axis is equal to unity. We denote finally by r_s and r_n the components of the radius-vector $\overline{OM}$ along the s- and n-axes.

A simple calculation shows that the condition of tangency of the velocity to the sheet, on its two faces, is given by the double relation

$$(\nu_n)_{ei} = (1 + \tilde{\omega}_{ei})r_n \cot \varphi \quad (5)$$

Taking (2) and (4) into account we have

$$\tilde{\omega}_e - \tilde{\omega}_i = \left[(\Phi_e - \Phi_i) - (\nu_{s_e} - \nu_{s_i})r_s - (\nu_{n_e} - \nu_{n_i})r_n\right]\cot \varphi$$

$$= (G - G'r_s) \cot \varphi - \delta r_n \cot \varphi$$

whence there is derived from (5)

$$\left.\begin{aligned} \tilde{\omega}_e - \tilde{\omega}_i &= (G - G'r_s) \frac{\tan \varphi}{r_n^2 + \tan^2 \varphi} \\ \delta = \nu_{n_e} - \nu_{n_i} &= (G - G'r_s) \frac{r_n}{r_n^2 + \tan^2 \varphi} \end{aligned}\right\} \quad (6)$$

We express the fact that the pressure is continuous across the sheet ($p_e = p_i$) by the Bernoulli law:

$$(1 + \tilde{\omega})(\tilde{\omega}_e - \tilde{\omega}_i) + \nu_s(\nu_{s_e} - \nu_{s_i}) + \nu_n(\nu_{n_e} - \nu_{n_i}) = 0$$

a relation that is finally put into the form

$$G - G' \frac{\nu_s(r^2 + \tan^2 \varphi) - (\Phi + \tan \varphi)r_s}{\nu_s r_s - (\Phi + \tan \varphi)} = 0 \qquad (7)$$

with

$$\Phi \equiv \Phi_i + G/2$$

$$\nu_s \equiv \nu_{s_i} + G'/2$$

$$r^2 \equiv r_s^2 + r_n^2$$

Equation (7) of our trumpet sheet is an integral-differential equation of a particular type and not classical as has hitherto been encountered. In considering for example $G(s)$ as the unknown function it enters through itself, its derivative, and integrals of the second order and of the first order expressing $\Phi_i(s)$ and $\nu_{s_i}(s)$ in terms of $G'(s)$ and $r(s)$.

Another unknown is the "shape" of the sheet; that is, the functions $r_s(s)$ and $r_n(s)$ or, if one prefers, the function $\zeta(\eta)$ which defines it in the ξ-plane. Besides equation (7) expressing the continuity of the pressure across the trumpet sheet, there are available of course the two equations (5) expressing the tangency of the flow of the upper and lower (prolonged) surfaces of this sheet. If one of these equations is regarded as defining by (4) the function $\delta(s)$, itself regarded as known from $G(s)$ and $\zeta(\eta)$ - or $r_s(s)$ and $r_n(s)$, which follows from $\zeta(\eta)$ - there is added to (7) only the remaining single equation (5), which is sufficient to determine the two unknown functions $G(s)$ and $\zeta(\eta)$ under the condition that the sources distributed in the ξ-plane outside the sheet are regarded as negligible, and hence that the totality of the compensating sources of the vortex sinks of the trumpet sheet are reduced to a single source, necessarily transferred to infinity. Here the sinks and compensating sources are each considered in the algebraic sense. Equation (5) can then be written

$$\nu_{ni} \equiv (1 + \tilde{\omega}_{ni})r_n \cot \varphi = \left[1 + (\Phi_i - \nu_{si}r_s - \nu_{ni}r_n)\cot \varphi\right]r_n \cot \varphi$$

Not to prolong the discussion unduly, I shall not here discuss the question, evidently an essential one, of knowing whether the solution of the complete equation (7) exists and whether it is unique, taking account of the fact that the flow becomes singular at infinity to which are transferred the compensating sources of the flow quantity fictitiously absorbed in the ξ-plane by the trumpet sheets.

I shall only remark that if one follows continuously the trace of the sheet in the ξ-plane starting from the leading edge A_1, for a wing at positive angle of attack: $G(s)$ constantly decreases but remains positive up to the crossing of the free edge of the sheet; $G'(s)$ is always negative, and at first small; $r_s(s)$, at first positive, vanishes very quickly - for the sheet is folded on itself over a very short space - then becomes negative; r_s changes moreover in sign each time that the radius vector $\overline{OM}$ becomes again normal to the trace of the sheet, that is, after each rotation of 180° of the directed tangent $\vec{s}$ to this trace.

Hence, at least for the initial portion of the sheet enclosed between A_1 and the tangent most to the left emanating from the origin O of the plane ξ (see fig. 8), the factor $(G - G'r_s)$ of δ is certainly positive; that is, δ has the sign of r_n, and is therefore negative. This means that this segment of the sheet at least is effectively represented, in a necessary manner, by a vortex-sink distribution.

If the sheet is further folded (i.e., if the rolling up into a trumpet is continued), the preceding character can reverse itself and this can be the case each time that the sign of r_n, or that of $(G - G'r_s)$, reverses. The totality of symmetric sheets, of course, relative to the two leading edges, and the finite area V of figure 7 should be equivalent to a sink absorbing the quantity that crosses the straits such as A_1A_1'.

POSSIBLE FORM OF EDGE OF TRUMPET-SHAPED SHEET

The assumed conicity - an assumption that could perfectly well be dropped - excludes the indefinite rolling up of the sheet on itself, since this arrangment would have to appear from the apex on.

We assume therefore that the rolling up is limited; that is, that the trumpet sheet presents a marginal edge, represented by a point B_1 of the ξ-plane (fig. 9) for the sheet of the right, emanating from the leading edge A_1. This sheet is then a conical fluid surface A_1B_1 at a pressure equal on its two faces at each point prolonging the surface of the wing and whose marginal edge B_1 is substituted in some way for the leading edge A_1 of the plane wing - extra slender and with sharp edges - here envisaged.

Since this sheet is formed to avoid the direct flowing around A_1 that would imply flow with continuous potential, it must be assumed, on the contrary, that the flow passes around the marginal border B_1 of this sheet. It will be noted incidentally that if it were not so, $G(s)$ and $G' = dG/ds$ would become zero at B_1, where one would then have zero intensity at the same time for the circulation and for the amount of flow through the trumpet-sheet surfaces.

The flow around B_1 by the pseudo crossflow is moreover evidenced by experience, which is here, according to the saying of Pascal, "the master whom we must follow."

It is not possible however to neglect the viscosity at the point B_1 nor in its neighborhood because it alone prevents the velocity from growing infinitely there, as would be the case for a perfect fluid flowing around the edge B_1 of our schematic sheet.

On account of the viscosity there is formed at B_1 a conical 'roll' progressively and continuously fed by the vortical strips that constitute the sheet and that wind over this sheet from the leading edge A_1, becoming finally, and sufficiently rapidly, oriented along the axis of the marginal roll when they reach it (see fig. 9).

In the case, depending on the angle of attack α, where the rolling-up angle of the sheet - total angle of rotation, in the ξ-plane, of the directed tangent $\vec{s}$ along the trace of the sheet - is less than π, the velocity lines of the pseudo crossflow are presented as schematized in figure 10, where the stagnation point A_e of figure 7 has been hypothetically placed on the positive part of the ζ-axis.

Let us consider this conception of the marginal roll of the trumpet sheet. It is a vortical zone where the viscosity of the real fluid, which cannot be neglected in this zone with very large velocity gradient, determines the carrying along by this roll of the fluid layers that come to pass around it. This passage around develops continuously as the distance from the apex increases, constantly thickening the vortical core, whose conical shape matches that of the trumpet sheet, according to our fundamental hypothesis of the conical affinity from the apex on, or conicity, for the entire flow.

The formation of the marginal roll can be represented, neglecting the viscosity, by the picture of figure 11. The vortical core (hatched area) absorbs, at its boundary, the discharge of the pseudo crossflow that has reached the strait A_1A_1' and that has not been absorbed through the trace A_1B_1' (in place of A_1B_1) of the trumpet sheet. One can even imagine that this core is with constant curl according to a conception recently developed by R. Legendre for a cylindrical roll (bubble) of the leading edge of a rectangular wing. We emphasize, at any rate, that, as the distance from the apex increases, the circulation of the marginal roll and its volumetric flow rate increase proportionally to the abscissa x, the first by the constant bringing up of strips of the trumpet-shaped vortex sheet (of reduced section emanating from A_1 but limited at B_1') and the second by the absorption of the flow through the lateral surface of the roll (according to the sink effect on the contour of the core in the ξ-plane).

The presence of this roll, its progressive development, its stability connected with that of the vortices in the interior of a fluid with negligible viscosity (outside these vortices) are very well corroborated by experience. In this way should be explained moreover the twists with multiple turns observed on visualized flow filaments and near the marginal rolls in a trumpet vortex sheet.

The same explanation applies in my opinion to the formation of the boundary vortices of rectangular wings or wings with very small sweepback and with almost elliptical lift distribution. At the marginal extremity of these wings, and starting from the leading edge itself, if it incurvates rapidly downstream in the marginal zone, a trumpet sheet is formed that produces its own marginal roll, and it is the latter, very concentrated and very durable, that constitutes the boundary vortex of the wing, in the usual sense.

PRACTICAL SCHEME OF FLOW

The rather difficult question remains to define from the foregoing (which is in direct accord with experience) a scheme such that it is practical; that is, such that it furnishes a sufficiently good approximation of the distribution of the velocities and pressures on and around the wing and that sufficiently simplifies the computations.

I have dwelt previously on the 'rational' conditions to which my conception of the trumpet-shaped sheet must restrict itself in order that the laws of the mechanics of fluids may be obeyed, with account taken of the intervention of viscosity both for explaining the separation of this sheet from the leading edge if it is sharp, or from a neighboring line if it is more or less rounded, and for explaining the formation of a vortical roll at the boundary of the sheet.

Among these rational conditions, there enter notably the pressure equilibrium at every point of the sheet and over its two faces, and a finite difference of the orientation presented at the sharp leading edge of a slender wing, by the velocity of the flow at the upper and lower surfaces.

I have thus emphasized the necessary theoretical connection of surface sinks (or sources) with the surface vortices of the trumpet sheet under the assumed hypothesis of conicity, and the approximate character of the concomitant hypothesis of solenoidal flux for the pseudo crossflow outside the preceding singularities. In recent times I have, with the collaboration of P. Duban, made several attempts at maximum simplification of the flow scheme. All those attempts where the trumpet sheet was only fragmentarily represented have furnished results which, while acceptable from several interesting points of view, involve some unsatisfactory local defect.

Another attempt, at present in the course of calculation, calls for a trumpet sheet with continuous vortex distribution from A_1 to B_1, completed at B_1 by a concentrated vortex sink (fig. 12). In spite of this forced 'stylization' of the true structure of the trumpet sheet, the computations are more cumbersome than the preceding. The results will be later presented if there is occasion.

In any case, the stylization (at B_1 and B_2) of the marginal vortex roll by a concentrated vortex sink, as I have indicated at Göttingen in 1953, appears to me to be imposed by the results of the visualization of the real flow and by a reasonable care of simplifying the computations, it being understood that it is a question of evaluating velocities and pressures on the wing and in its immediate neighborhood.

TEARING OF THE TRUMPET SHEET

It is evident that in reality and for a delta wing of large chord the conical flow is only approximate and only for a limited portion of the wing.

The viscosity exerts in fact a cumulative influence along the leading edge and from the apex, as regards the formation and separation of the trumpet sheets. This influence causes a progressive divergence of the real flow from the affine flow, in an approximate manner, for a perfect fluid. It may therefore be considered as probable that at a certain distance from the apex the trumpet sheet, formed from this point, tears at the leading edge and a new piece of trumpet sheet is formed from this tearing point on, terminating in its turn in a more downstream point on the leading edge, and so on.

Thus, several rolls, kinds of concentrated vortices, can appear above and along a delta wing, which then roll up more or less rapidly the one about the other.

The effect of the interaction of the trailing edge on the flow about the leading edge can similarly favor the tearing of a trumpet sheet before its development reaches the boundary point of the delta wing.

Numerous observations of visualized flows conducted at the O.N.E.R.A., on slender delta wings and wings with strong backsweep as well as on spindle-shaped bodies, appear to me to justify the preceding conception, which I here restrict myself to mentioning, emphasizing the effect, evidently of essential importance, of the Reynolds number relative to the 'depth' of the obstacle in the flow direction.

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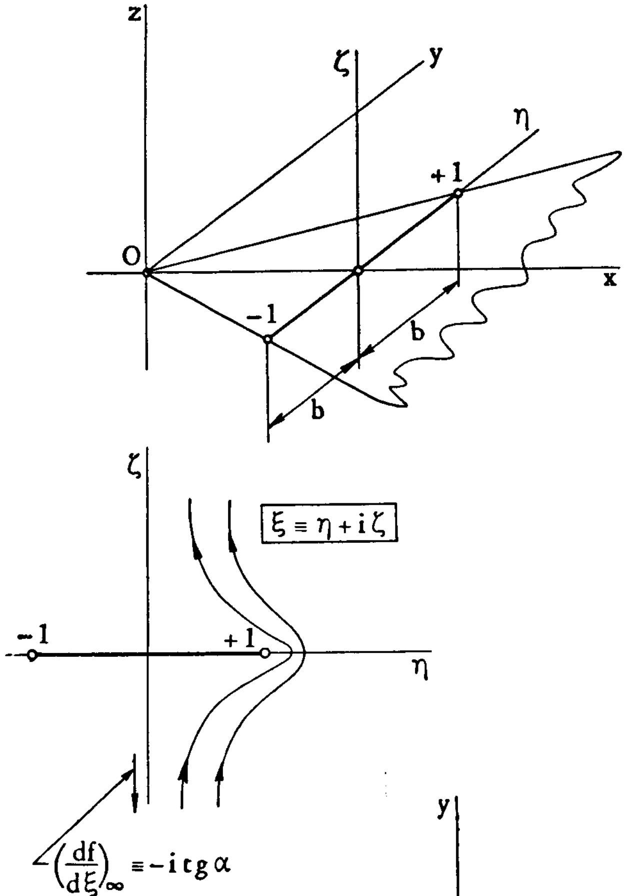
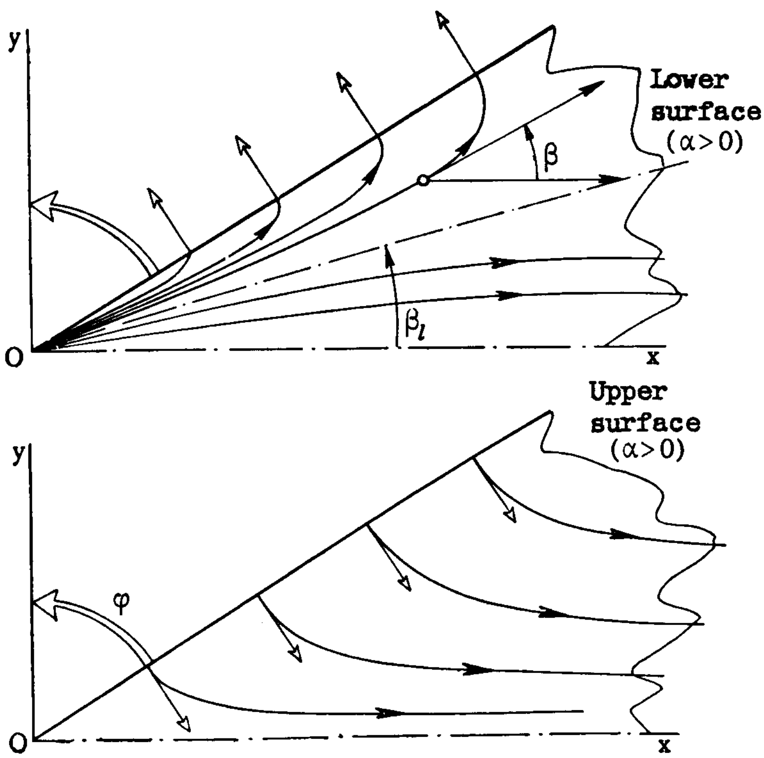
6. Brown, C. E., and Michael, W. H.: Jour. Aero. Sci., Oct. 1954, p. 792.

Translated by S. Reiss
National Aeronautics and
Space Administration



z
ζ
y
η
+1
O
x
-1
b
b
ζ
$\xi \equiv \eta + i\zeta$
-1
+1
η
$\left(\frac{df}{d\xi}\right)_\infty \equiv -i\,tg\,\alpha$

Fig. 1.



y
Lower
surface
(α>0)
β
β_l
O
x
Upper
surface
(α>0)
y
φ
O
x

Fig. 2.

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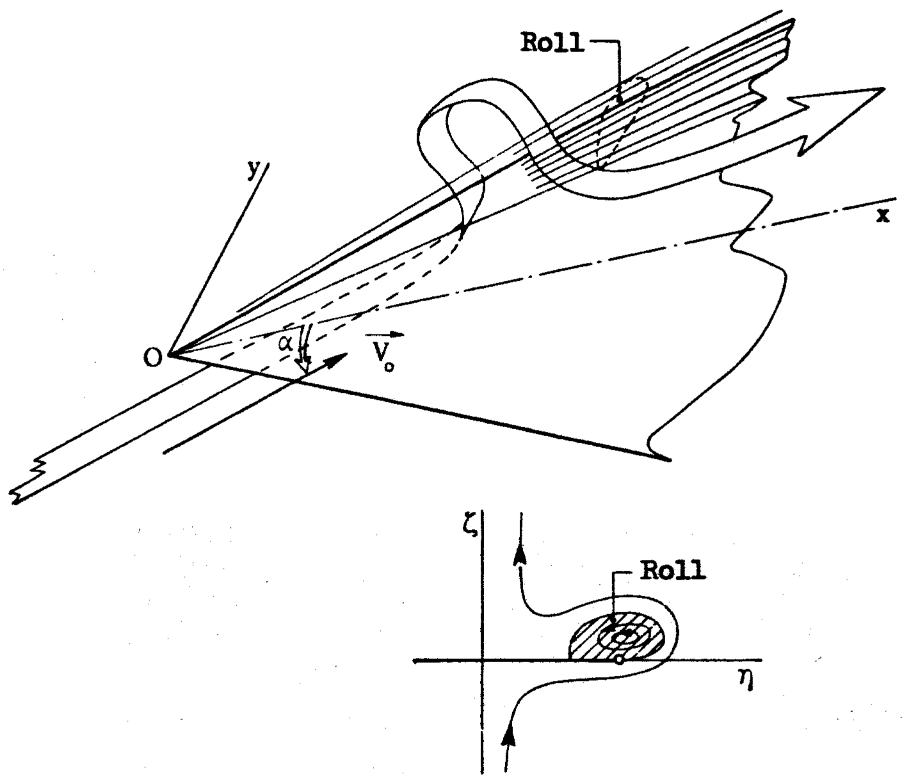


Fig. 3.

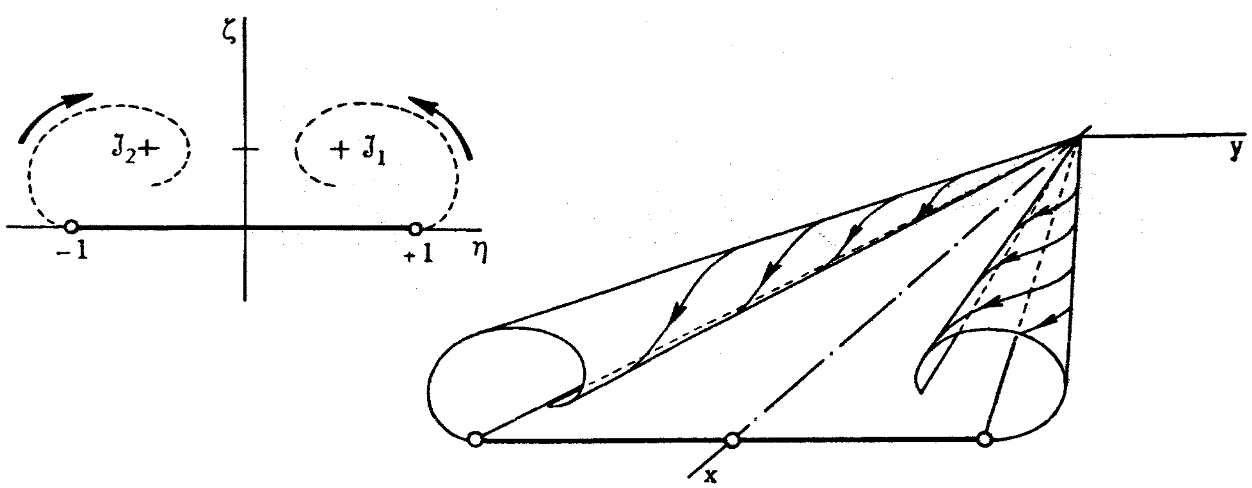


Fig. 4.

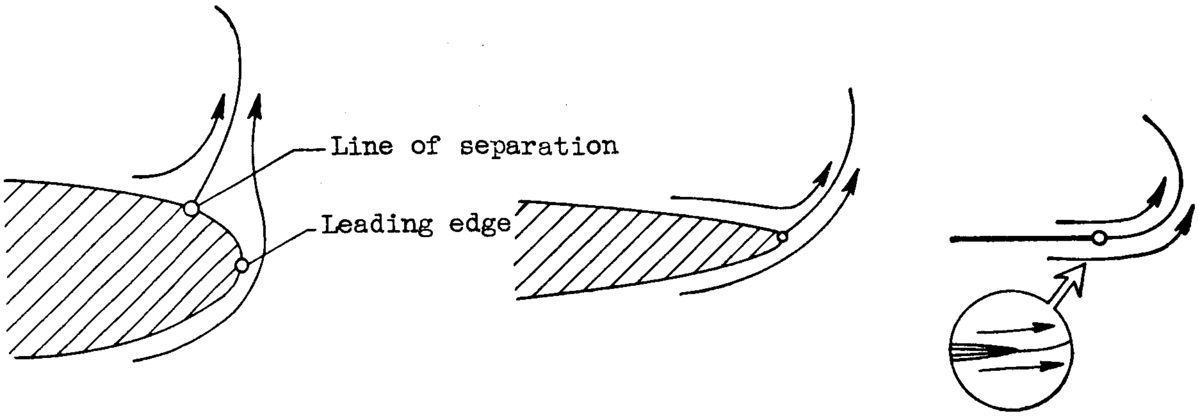
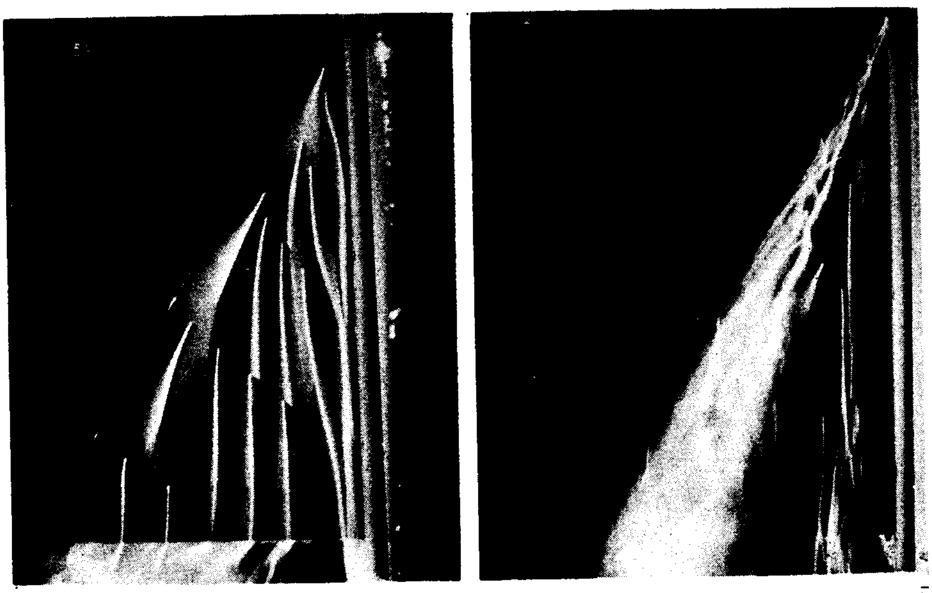


Fig. 5.



Lower surface (α = 20°) Upper surface (α = 11°)

Fig. 6.

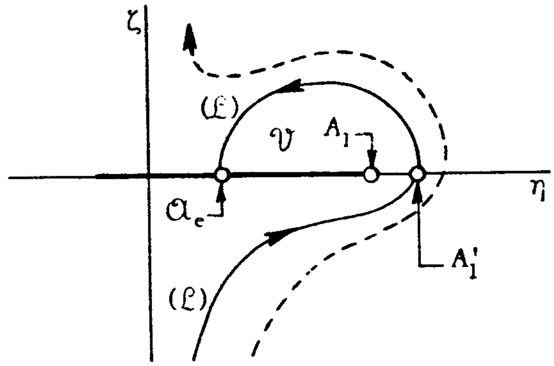

ζ
(£)
ϑ
A_1
η
$\mathfrak{a}_e$
A_1'
(£)

Fig. 7.

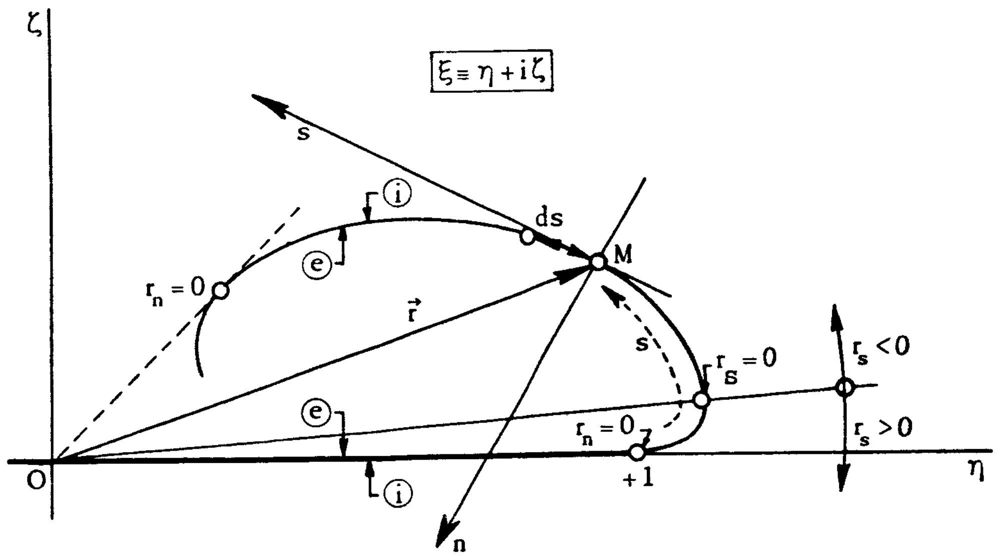

ζ
$\xi \equiv \eta + i\zeta$
s
i
ds
e
M
$r_n = 0$
$\vec{r}$
s
$r_s = 0$
$r_s < 0$
e
$r_n = 0$
$r_s > 0$
O
+1
η
i
n

Fig. 8.

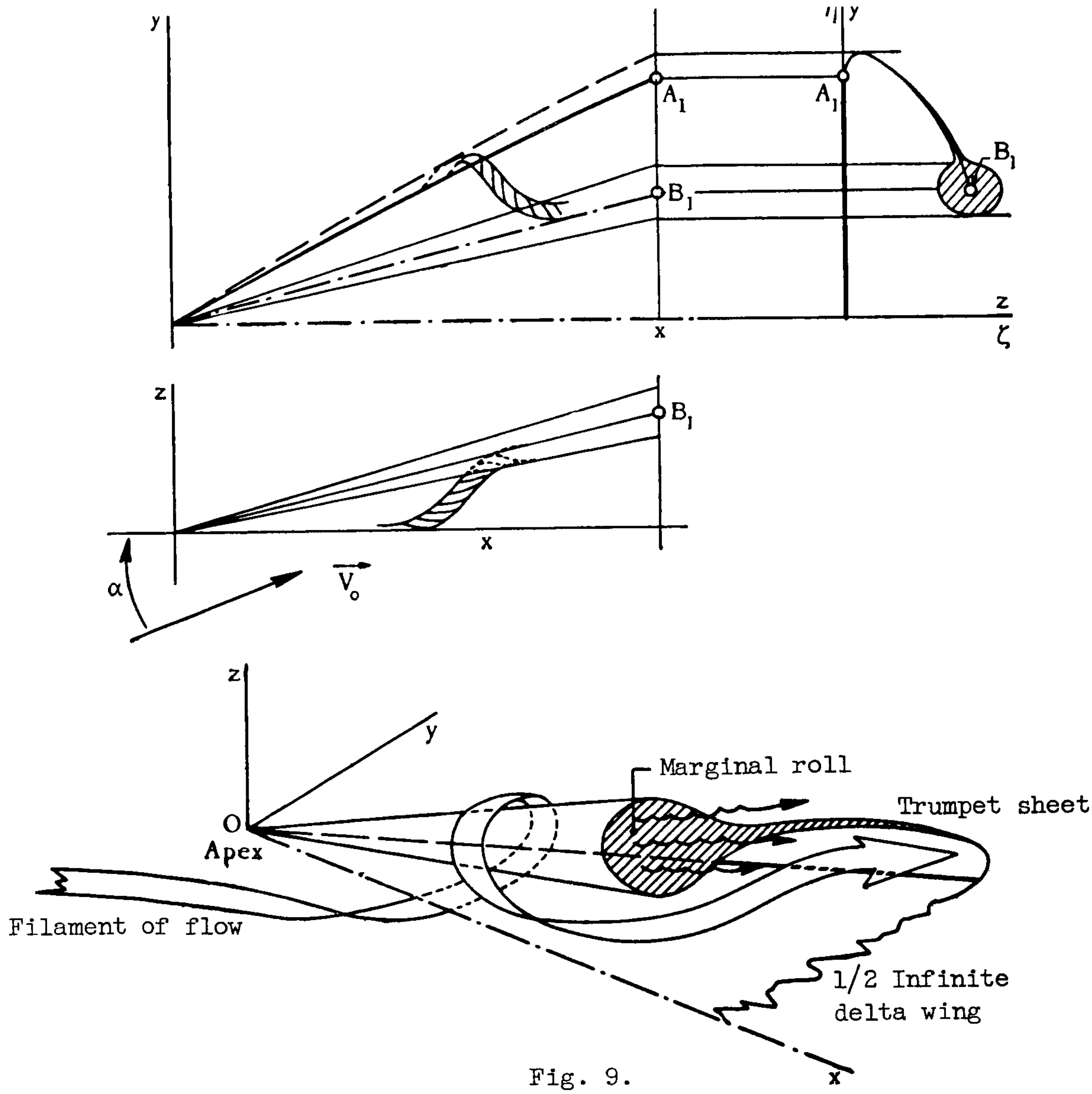

y
A1
A1
η
y
B1
B1
z
x
ζ
z
B1
x
α
V0
z
y
O
Apex
Marginal roll
Trumpet sheet
Filament of flow
1/2 Infinite
delta wing
x

Fig. 9.

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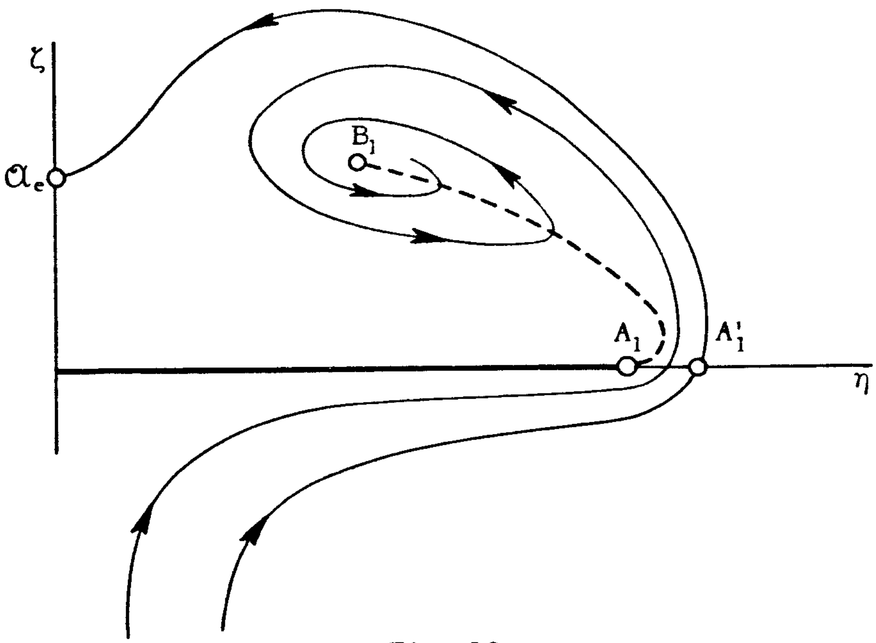

Fig. 10.

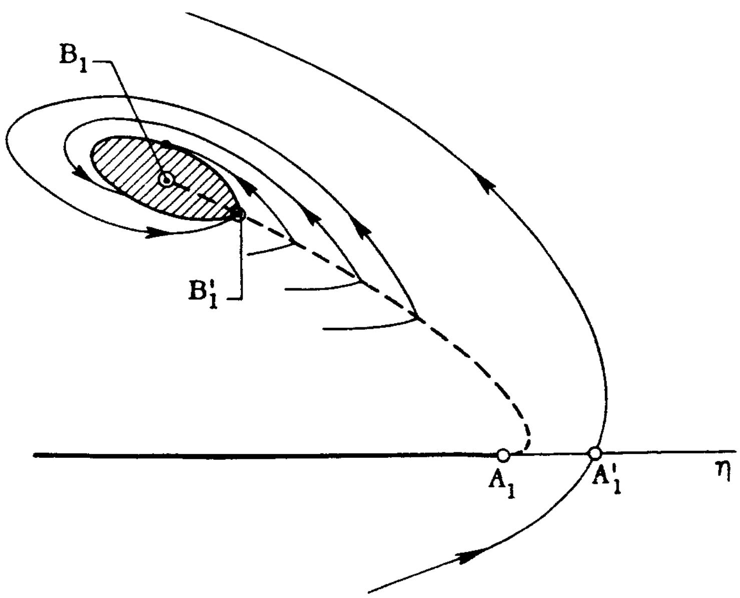

Fig. 11.

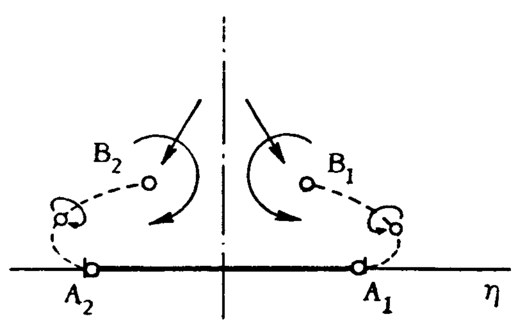

Fig. 12.